Indian Statistical Institute
M. Math. I year
Second Semester Examination 2010
Topology II

Date: 30-4-2010 Marks: 45 Instructor: Jishnu Biswas

Attempt all questions. If you use a result proved in class in your solution, please quote in full and clearly. Total marks - 45

1) True or false? If true give reasons, and if false give an example.

a) If two topological spaces are homeomorphic then they have isomorphic singular homology groups. (3 marks)

b)If two topological spaces have isomorphic singular homology groups, then they are homeomorphic. (3 marks)

c) Let $A \subset X$ be a path connected subspace of a path connected topological space. Then the homomorphism induced by the inclusion $i : A \to X$ at the level of the zeroth homology groups $i_* : H_0(A) \to H_0(X)$ is the identity map. (3 marks)

d) Same question as (c) for the induced map $i_* : H_1(A) \to H_1(X)$. (3 marks)

2) What is a Δ-complex structure on a topological space X? Define a Δ-complex structure on the real projective plane $\mathbb{R}P^2$ and compute all simplicial homology groups $H_i^{\Delta}(\mathbb{R}P^2)$ for all $i \geq 0$ with respect to this Δ-complex structure. (6 marks)

3) What is a good pair (X, A)? Write down the long exact homology sequence of a good pair (X, A). Use it to derive all the homology groups of spheres of all dimensions. (6 marks)

4) What is the Mayer-Vietoris sequence? Use it to compute all the singular homology groups of a torus. (7 marks)

5) Let X be the topological space which is a wedge of a two dimensional sphere (S^2) with two circles (S^1's) at two different points. Compute all the singular homology groups of this space. (7 marks)

6) State the excision theorem. Let I be a homeomorphic copy of the closed interval $[0, 1]$ in the sphere S^2, and let P be a point on I. Does the conclusion of the excision theorem hold for the triple $(S^2, I, \{P\})$? (7 marks)